

The Pioneer Anomaly

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OBJECTIVE

The purpose here is to examine the Pioneer Anomaly, especially in view of the insights provided by gravimagnetics.¹

THE PIONEER ANOMALY DEFINED

The solar mass, or more specifically ($G M_{\text{sun}}$) is determined based on orbital mechanics. However, orbital mechanics near the sun are affected by gravimagnetic fields, solar wind, and possibly gravitational shadowing from the sun itself, dark matter, and cosmic matter. Orbital mechanics close to the sun that work for one estimate of ($G M_{\text{sun}}$) may fail when the sun is departed by the distances traveled by Pioneer 10 and 11. Pioneer 10 and 11 may thus be useful for measuring any error in ($G M_{\text{sun}}$).

The Pioneer anomaly can be characterized as an unexplained frequency drift^{2,3} of size $(5.99 \pm 0.01) \times 10^{-9}$ Hz/s.

The Pioneer 10 and 11 spacecraft used S-band frequencies, transmitting at frequency $f = 2.29$ GHz.⁴ This gives wavelength:

$$\lambda = c / f = 0.1309 \text{ m.}$$

At 26.36 AU Pioneer 10 experienced $a_{\text{sun}}(r) = 8.5343 \times 10^{-6} \text{ m/s}^2$ while the value of a_{const} corresponding to the frequency drift error was $(8.68 \pm 0.50) \times 10^{-10} \text{ m/s}^2$, almost exactly one part in 10,000. This, taken only by itself, could be explained by an 0.01 percent error in estimating ($G M_{\text{sun}}$).

However, at 45.7 AU, Pioneer 10 experienced $a_{\text{sun}}(r) = 2.8394 \times 10^{-6} \text{ m/s}^2$ while the value of a_{const} corresponding to the frequency drift error was $(8.24 \pm 0.20) \times 10^{-10} \text{ m/s}^2$, about exactly 2.9 parts in 10,000. The percent error increases with distance. In fact, $(45.7 \text{ AU} / 26.36 \text{ AU})^2 = 3$, which is tantalizingly close to the observed factor 2.9. This intuitively then seems to indicate an error in the determination of r .

FREQUENCY DRIFT

Given speed of light c , departure velocity v and transmission frequency f , the doppler

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shift provides an observed frequency f_{obs} :

$$f_{\text{obs}} = f + f(v/c) = f + (1/\lambda) v$$

In an environment with acceleration $dv/dt = a(r)$ at radius r we can differentiate f_{obs} with respect to t in order to obtain the expected frequency drift $f_{\text{drift}} = df_{\text{obs}}(r) / dt$:

$$f_{\text{drift}} = df_{\text{obs}}(r) / dt = d(f + (1/\lambda) v) / dt = (1/\lambda) dv / dt = (1/\lambda) a(r)$$

The acceleration due to the sun's gravity is:

$$a_{\text{sun}}(r) = (G M_{\text{sun}}) / r^2$$

so, substituting:

$$f_{\text{drift}} = (1/\lambda) a(r) = (1/\lambda) (G M_{\text{sun}}) / r^2$$

Note that we can use the above, given the frequency drift rate $f_{\text{drift}} = df_{\text{obs}}(r) / dt$, and transmission wavelength λ , to determine distance r :

$$r = [(1/\lambda) (G M_{\text{sun}}) / f_{\text{drift}}]^{1/2}$$

Further, we should expect the distance r to the Pioneer spacecraft, if not derived from this expression for r , to be consistent with it. However, suppose our estimate of $(G M_{\text{sun}})$ is off by a factor Z_1 very near 1. We then have the r resulting from this computation based on f_{drift} , off by a factor Z :

$$Z = (1/Z_1)^{1/2}$$

When we subsequently calculate, here using $(G M_{\text{sun}})$ to be the true value, and r the true radius:

$$a_{\text{sun}}(r) = (G M_{\text{sun}}) Z_1 / (r Z)^2 = (G M_{\text{sun}}) Z_1 / (r^2 1/Z_1)$$

$$a_{\text{sun}}(r) = (G M_{\text{sun}}) Z_1^2 / r^2$$

If our estimate of the sun's mass is just a bit too high by a factor Z_1 , then the estimate for the expected acceleration at r will be too high by a constant factor of Z_1^2 . From the data we can see this is not what happens. At great distances, the

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acceleration appears to be off by an approximately constant value, not a constant factor, as shown in Table 7 - Pioneer 10-11 Anomalous Acceleration Data.

r (AU)	date (yr)	alpha (E-10 m/s ²)	a = MG/r ² (E-6 m/s ²)	v = dr/dt (m/s)
Pioneer 11 (Saturn slingshot):				
9.39	80.1970	1.56	-67.20	
12.16	82.5213	6.28	-40.07	5650
14.00	83.4351	8.05	-30.23	9546
16.83	84.6951	8.15	-20.92	10648
18.90	85.5665	9.03	-16.59	11262
22.25	86.9428	8.13	-11.97	11539
23.30	87.3708	8.98	-10.91	11629
26.60	88.7020	8.56	-8.37	11752
29.50	89.8662	8.33	-6.81	11809
			Terminal v:	10450
Pioneer 10 (Jupiter slingshot):				
26.36	82.0520	8.68	-8.53	
28.88	82.9510	8.88	-7.10	13288
31.64	83.9469	8.59	-5.92	13138
34.34	84.9264	8.43	-5.02	13068
35.58	85.3777	7.67	-4.68	13026
37.33	86.0178	8.43	-4.25	12960
40.59	87.2189	7.45	-3.60	12866
43.20	88.1861	8.09	-3.17	12793
45.70	89.1158	8.24	-2.84	12748
			Terminal v:	11322

Table 7 - Pioneer 10-11 Anomalous Acceleration Data

Columns 1-3 demonstrate the anomaly. Column 3, “alpha”, is the unexplained acceleration magnitudes corresponding to the distances r given in AU. Column 4 shows the sun’s gravitational field acceleration value at the given distance r, and

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Column 5 is an approximation of dr/dt , a velocity defined here to be in the r direction, computed by dividing the incremental distances between successive entries in Column 1 by the incremental time from successive entries Column 2. JPL values for the terminal velocities (at $r = \text{infinity}$) are included also in Column 5. Note that the acceleration values α are actually negative, i.e. directed at the earth and reducing velocity v with time.

It is most interesting that the anomalous acceleration is roughly proportional to v . It is therefore reasonable here to hypothesize the anomaly is caused by a gravimagnetic Lorentz force due to the ambient gravimagnetic field from the galaxy.

THE GRAVIMAGNETIC LORENTZ FORCE EXPLANATION

Given the EM Lorentz force:

$$F = q (v \times B)$$

we have the GK equivalent:

$$F_g = (i m) (v \times K)$$

$$F_g / m = a_{\text{Lorentz}} = i (v \times K)$$

Now comes a critical part of the hypothesis. It is necessary to explain how an apparent acceleration results in the radial direction, i.e. the r direction, even at great distances where the viewing angle is zero, i.e. directly along the vector r . At great distances, the velocity vector v and the distance vector r approach being parallel, i.e. co-aligned. How can a Lorentz force cause an acceleration along the direction of r and v ? The answer is that the Lorentz force can not add or subtract energy from the satellite. The Lorentz force, when it deflects the velocity vector v of the satellite, diminishes the remaining component of v along the vector r . Now, the measurement methods used by JPL very accurately determine, even at great distances, both the doppler shift (change in v along vector r), and the magnitude of r itself. At great distances, however, the corresponding small changes in tangential velocity can not readily be directly measured.

A remaining problem is the fact that a tangential component to v can potentially result in an acceleration in the r direction, which will therefore be measurable even

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at great distances. This tangential velocity component, though present for both Pioneer 10 and 11 tracks, does not produce a large acceleration. There is a solution to this problem. The flight paths of Pioneer 10 and 11 are almost collinear, but in opposed directions. Pioneer 10 is headed toward 4h 35 m RA, + 16 deg. dec., while Pioneer 11 is headed roughly at 17 h 45 m RA, -29 deg. dec., about 13 h, or about 195 deg. apart. We thus might expect the ambient gravimagnetic field to be pointing in the general direction of 10h 38m RA, 7 deg. dec. In this way the tangential Lorentz force is directed orthogonal to the plane of the orbital, and thus results in small changes in the orbital parameters without affecting the doppler shift. Such a specific alignment seems unlikely, and is not right, as we shall see, though it does on first inspection seem to explain the observed results.

COMPUTING THE LORENTZ FORCE

The observed anomaly values let us approximate the magnitude of the ambient gravimagnetic field. From:

$$a_{\text{Lorentz}} = i (\mathbf{v} \times \mathbf{K})$$

and with \mathbf{v} approximately normal to \mathbf{K} , we have scalar:

$$a = i (\mathbf{v} \cdot \mathbf{K})$$

$$i \mathbf{K} = a / \mathbf{v}$$

The problem we have now is to estimate the acceleration on the satellite given the acceleration measured in the direction of \mathbf{r} . Fig.2, Velocity Vectors, provides a diagram of the situation.

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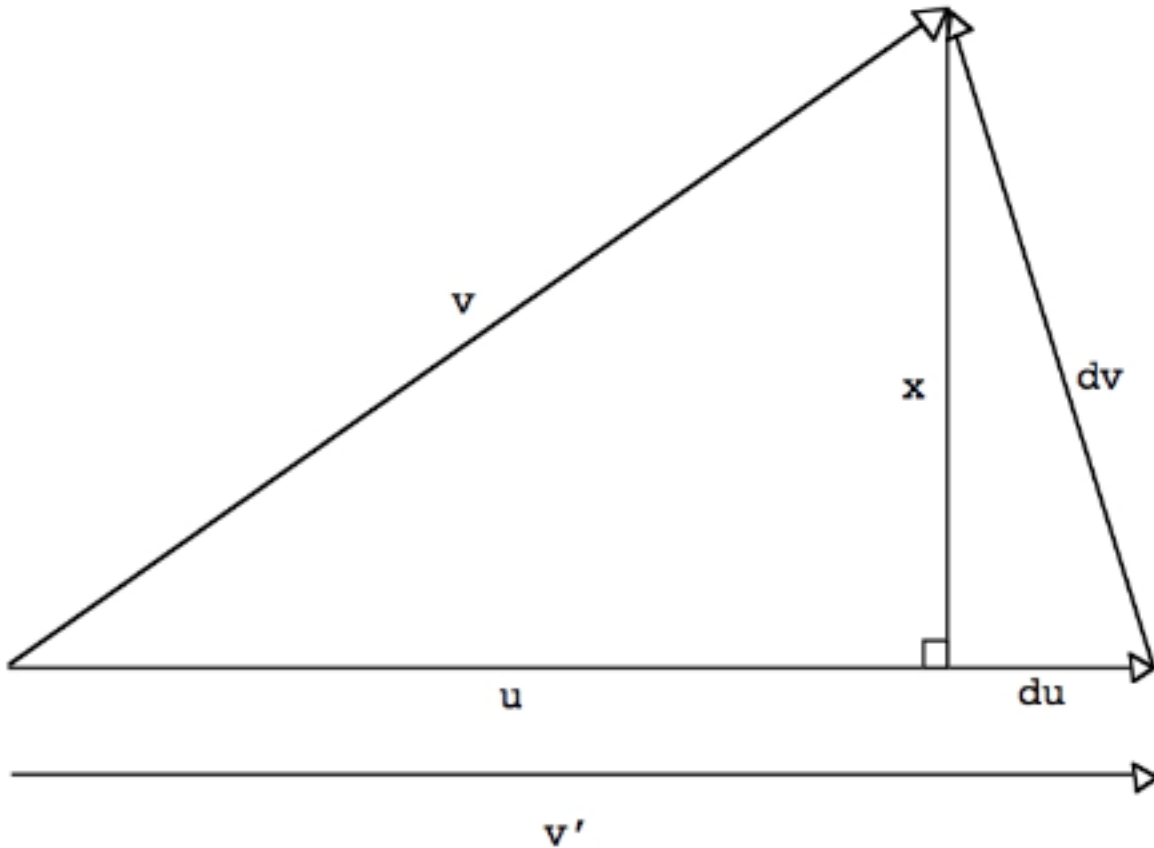


Fig. 1 - Velocity Vectors

In Fig. 2, we see that a satellite with original velocity vector v' is changed to vector v , giving velocity change dv . In the direction of the original velocity vector v' , a deceleration du is experienced, leaving velocity in that axis u . Since energy is conserved, we have the magnitude of v' equal to the magnitude of v , and $v' = u + dv$.

Now considering only magnitudes, we have:

$$u = v - dv$$

and squaring:

$$u^2 = v^2 - 2 v dv + (dv)^2$$

From the large triangle:

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$$x^2 = v^2 - u^2 = v^2 - (v^2 - 2 v du + (du)^2) = 2 v du - (du)^2$$

From the small triangle:

$$(dv)^2 = x^2 + (du)^2 = (2 v du - (du)^2) + (du)^2$$

$$(dv)^2 = 2 v du$$

$$dv = (2 v du)^{0.5}$$

For Pioneer 10 at 45.7 AU we have an acceleration of $-8.24 \times 10^{-10} \text{ m/s}^2$, and velocity v of 12748 m/s. This gives a per second velocity change du of $-8.24 \times 10^{-10} \text{ m/s}$, and thus

$$dv = (2 (12748 \text{ m/s}) (8.24 \times 10^{-10} \text{ m/s}))^{0.5}$$

$$dv = 4.58 \times 10^{-3} \text{ m/s}$$

We thus have a satellite acceleration $a = 4.58 \times 10^{-3} \text{ m/s}^2$ with which to compute K :

$$i K = a / v = (-4.58 \times 10^{-3} \text{ m/s}^2) / (12748 \text{ m/s}) = - 3.59 \times 10^{-7} \text{ Hz}$$

$$K = 3.59 \times 10^{-7} i \text{ Hz}$$

Similarly, for Pioneer 11 at 29.5 AU we have:

$$dv = (2 (11809 \text{ m/s}) (8.33 \times 10^{-10} \text{ m/s}))^{0.5}$$

$$dv = 4.44 \times 10^{-3} \text{ m/s}$$

$$i K = a / v = (-4.44 \times 10^{-3} \text{ m/s}^2) / (11809 \text{ m/s}) = - 3.75 \times 10^{-7} \text{ Hz}$$

$$K = 3.75 \times 10^{-7} i \text{ Hz}$$

It is notable that a consistent discrepancy between the values of K for the two satellites, though small, might be resolved by assuming our solar system has a velocity component (here, of about 500 m/s) in the axis of satellite motion through the ambient gravimagnetic field. It is also notable that K as estimated here unfortunately appears to be much too large to be unnoticed in solar system

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mechanics. However, the best fit K field direction and magnitude is undoubtedly not that chosen, but rather more polar, the north pole pointing in the direction of Draco, and the line of sight r is not aligned with v' thus the acceleration on the r axis varies depending on earth location. The best fit value of K is expected to be much smaller when modeling of all orbital characteristics and track correction methods is incorporated. The value of the doppler shift, especially the value of any tangential velocity contributions to a , depends on the direction of r , and thus the angle of observation, and thus the location of earth in its orbit. Therefore, the anomalous accelerations are expected to have an annually varying component, which they do. This strongly supports a celestial polar alignment for the gravimagnetic field. The total Lorentz force along r depends much more on the large dv than the tiny du . K thus has a lower bound given by:

$$K = i a / v = i (8.33 \times 10^{-10} \text{ m/s}^2) / (11809 \text{ m/s}) = 7.05 \times 10^{-14} \text{ i Hz}$$

It is expected the best empirical fit for the value of K will be near this value, as the axis of the ambient gravimagnetic field north should be approximately aligned with Draco. Note that the earth's gravimagnetic field 1.815×10^{-13} (i Hz) may be similar in magnitude, and is aligned 23 degrees from the celestial pole in Draco, thus the two fields reinforce within the earth, but oppose with regard to satellites, at high altitudes. The earth's field lorentz force on a west to east satellite reduces its attraction toward earth, while the ambient gravimagnetic field increases its attraction, just as it increases the attraction of the Pioneers to the sun. It is expected, depending on latitude, there will be an apparent diurnal fluctuation of the earth's gravimagnetic field due to nonalignment with the ambient field.

SUMMARY

An underlying cause for the Pioneer Anomaly has been proposed: a Lorentz force from the ambient gravimagnetic field of the galaxy. This force has the desirable characteristics in that it remains relatively constant with distance from earth, diminishes long range as satellite velocity diminishes, and produces small annual doppler variations. Verifying the galactic gravimagnetic field as the cause of the Pioneer anomaly requires a complete modeling of all orbital characteristics and track correction methods, taking into account the existence of gravimagnetics.

¹ Horace Heffner, 2007, "Gravimagnetics",
<http://mtaonline.net/~hheffner/FullGravimag.pdf>

² Pioneer Extended Mission Plan, Revised, NASA/ARC document No. PC-1001
(NASA, Washington, D.C., 1994)

³ Michael Martin Nieto and John D Anderson, 2005, "Using Early Data to
Illuminate the Pioneer Anomaly", *Class. Quant Grav.* 22 (2005) 5343-5354,
<http://arxiv.org/abs/gr-qc/0507052v2>

⁴ Anderson et al, 2001, "Study of the Anomalous Acceleration of Pioneer 10 and
11", *Phys. Rev. D* 65 (2002) 082004, p 5,
<http://arxiv.org/abs/gr-qc/0104064v5>